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Effect of porosity on the Rippling motionof Herschel-Bulkley fluid in a non uniform tube

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---***ABSTRACT***---
The consequence of varying pressure drop on a Herschel-Bulkleyfluid through porous tube has been explored. The governing equations of the model areanalytically solved under long wavelength and small Reynolds number approximations.Results areobtainedfor the parameters: Darcy number, slip parameter, yield stress, axial radius on the pressure difference. Theobservationsdisclose that the building up of the porous thickening of the wall enhances the pressure. Also, rise in Darcy numbersreduces the pressure rise.
KEYWORDS: *Peristalsis, Porous tube, Pressure drop, Slipparameter*

I. INTRODUCTION

Peristaltic motion is the random movement of the muscles of the channel creating wave motions that drive the contents of the canal frontward as they travel. In the digestive tract of human being, gastrointestinal tract, etc., smooth muscle tissues contract in succession,that generate a peristaltic wave propellingthe food forward. Earthworms and some modern machinery use a similar mechanism for their locomotion.

Latham[1] was the principal investigator who studied the motion of a fluid in a peristaltic pump. The lubrication theory model wherein the model is considered to be inertia free and wall curvature is negligible, as discussed by Jaffrin and Shapiro[2]. Mekheimer and Arabi[3]examined the peristaltic flow inside a porous medium with MHD effect.

A non-Newtonian fluid does not follow Newton's law of viscosity. Numerous molten polymers and salt solutions are non-Newtonian fluids.Lately, the use of non-Newtonian fluids have become almost an essentialneed for the engineering, industrial systems and medical science. Therefore theresearch of diverse scientific mechanism with non-Newtonian fluids has also expanded.Medhavi[4] studied ripplingflow of non-Newtonian fluid.Eldabe and Abou-Zeid[5]have analyzedmass and heat transfer ofnon-Newtonian fluid througha channelunder rippling. Sankad and Patil[6] havestudied the non-Newtonian fluid flow inside a non-uniformconduit lined with porous peristaltic material.

The pores are naturally packed with a fluid (liquid or gas).Eldabe et al.[7] has examinedthe non-Newtonian fluid flowing inside the horizontal channel under magnetic effect. Sankad and Dhange[8] did analysis of the incompressible viscous fluid transport inside a peristaltic medium having pores, underchemical reactions and wall effects.Sankad and Nagathan[9] investigated the heat transfer and slip impactsof MHD couple stress fluid through rippling motionunder porosity.

In uniform tube the cross section of each stream of the tube remains unchanged and every particle progresses along its axis with constant speed. Srinivasacharya et al.[10]consideredmicropolar fluid transport through ripplingpipe.Vajravelu et al[11]inspected the pumping of a Casson fluid in peristaltic tube having elasticity.Selvi et al.[12] analyzed peristaltic transfer of power-law fluid inside a flexible tube.

In a Herschel–Bulkley fluid model, the strain of the fluid is non-linearly associated to the stress. Vajravelu et al.[13]analysed the rippling transportation of a Herschel-Bulkley fluid in a flexible tube. RajashekharChoudhari et al.[14]discussedripplingmotion of Herschel-Bulkley fluid in a flexible tube having porous walls under slip condition.

II. MATHEMATICAL FORMULATION

Herschel-Bulkleyfluid flow inside a non uniformcirculartube is considered under peristaltic motion with coordinatesystem(r, z, t). The blood flow is modeled to be laminar, steady, incompressible, two-dimensional, axisymmetric and exhibiting peristaltic motion of Herschel-Bulkley fluid in an elastic tube of radius. The region between$r = 0$ and $r = r_0$ is called as plug flow region where $|\tau_{rz}| \le \tau_{0}$. in the region between$r = r_0$ and $r = a(z, t)$, we have$|\tau_{rz}| \ge \tau_{0}$.

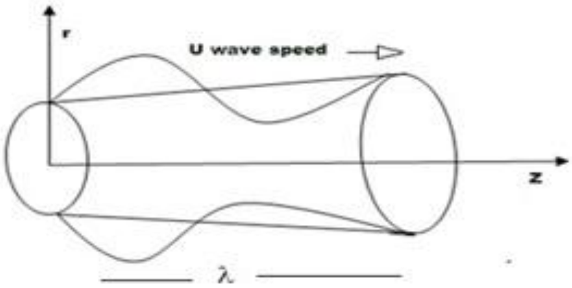

Fig. 1.1Geometry of a symmetric peristaltic non-uniform tube with porous wall

The deformationof walldue to the transmissionof rippling wavesis represented as

$$H(z,t) = a_0 + b\sin\frac{2\pi}{\lambda}(z-ct), \quad (1)$$

where$a_0 = a + dz$ anda_0: the mean radius of the tube;b: the amplitude of the wave; λ:is the wavelength;and$c$:the wave speed.
The constitutive equation for Herschel-Bulkley fluid is

$\tau = \mu(\dot{\gamma})^n + \tau_0$for $\tau \geq \tau_{0,}$

$\dot{\gamma} = 0$ for $\tau < \tau_{0.}$

Governing equations

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = -\frac{\partial p}{\partial z}, \quad (2)$$

$$\frac{\partial p}{\partial r} = 0, \quad (3)$$

whereτ_{rz} is shear stress,

$$\tau_{rz} = \mu\left(-\frac{\partial u}{\partial r}\right)^n + \tau_{0.}(4)$$

Here, n: the power law index;τ_0: the yield stress of the tube.
The dimensionless quantities are

$$\bar{r} = \frac{r}{a_0}, \bar{z} = \frac{z}{\lambda}, \bar{u} = \frac{u}{U}, \bar{q} = \frac{q}{\pi a_0{}^2 U}, \quad \bar{Q} = \frac{Q}{\pi a_0{}^2 U}, \quad \bar{a} = \frac{a}{a_0}, \bar{p} = \frac{a_0{}^{n+1}}{\lambda\mu U^n}P,$$

$$\bar{\tau_0} = \frac{\tau_0}{\mu\,(U/a_0)^n}, \quad \overline{\tau_{rz}} = \frac{\tau_{rz}}{\mu(U/a_0)^n}, \quad \bar{r_0} = \frac{r_0}{a_0}.$$

The Eq. (2) is solved using the boundary conditions:

$$\psi = 0 \ at r = 0, \quad (5)$$

$$\psi_{rr} = 0 \ at r = 0, \quad (6)$$

$$\tau_{rz} = 0 \ at r = 0, \quad (7)$$

$$u = -\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial r} - 1 \quad atr = h(z) - \epsilon. \tag{8}$$

Here,u is velocity, Da: the Darcy number;α: the slip parameter andϵ: the porous thickening of the wall.

III. SOLUTION OF THE PROBLEM

Using$p = -\frac{\partial p}{\partial z}$ in Eq. (2) we get $\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) = p.$

Using Eqs. (5-8), Eqs. (2) and (4) are solved for the velocity field:

$$u = \left(\frac{p}{2}\right)^m \left[\frac{1}{m+1}((k)^{m+1} - (r - r_0)^{m+1})\right] + \frac{\sqrt{Da}}{\alpha}(k)^m - 1, \tag{9}$$

where$k = h - \in - r_0$,$m = \frac{1}{n}$.

Using $\frac{\partial u}{\partial r} = 0$ at$r = r_0$, the superior limit of the plug flow region is given by$r_0 = \frac{2\tau_0}{p}$ and also using the condition $\tau_{rz} = 0$ $atr = 0$we obtain$p = \frac{2\tau_0}{r_0}$.

Hence

$$\frac{r_0}{a} = \frac{\tau_0}{\tau_a} = \tau, \qquad 0 < \tau < 1. \tag{10}$$

Using (10) along with $r = r_0$ in Eq. (9), the plug flow velocity is

$$u_p = \left(\frac{p}{2}\right)^m \left\{\left[\frac{1}{m+1}(k)^{m+1}\right] + \frac{\sqrt{Da}}{\alpha}(k)^m\right\} - 1 \quad for \quad 0 \le r \le r_0. \tag{11}$$

Integrate Eqs. (9) and (11) w.r.t$'r'$and $\psi_p = 0$ $atr = 0$,$\psi = \psi_p$ at $r = r_0$we obtain the stream functions as

$$\psi = \left(\frac{p}{2}\right)^m \frac{1}{m+1}(k)^{m+1} r - \left(\frac{p}{2}\right)^m \frac{1}{m+1}\frac{(r - r_0)^{m+2}}{m+2} + \frac{\sqrt{Da}}{\alpha}(k)^m r - r, \tag{12}$$

$$\psi_p = \int u_p dr = \left(\frac{p}{2}\right)^m (k)^m \left[\frac{k}{m+1} + \frac{\sqrt{Da}}{\alpha}\right] r - r. \tag{13}$$

The volume flux q:

$$q = \int_0^{r_0} u_p r dr + \int_{r_0}^{a} u r dr,$$

$$q = \left(\frac{p}{2}\right)^m \left\{\frac{a^2}{2}(k)^m \left[\frac{k}{m+1} + \frac{\sqrt{Da}}{\alpha}\right] - \frac{(a - r_0)^{m+2}}{(m+1)(m+2)}\left[a - \frac{(a - r_0)}{(m+3)}\right]\right\} - \frac{(a^2 - r_0^{\ 2})}{2}. \tag{14}$$

From Eq. (14) will get

$$p = -\frac{\partial p}{\partial z} = 2\left[\frac{(2q + a^2 - r_0^{\ 2})s}{a^2(k)^{m+1}(m+2)(m+3) + \frac{\sqrt{Da}}{\alpha}a^2(k)^m s - 2a(a - r_0)^{m+2}(m+3) - 2(a - r_0)^{m+3}}\right]^{1/m},$$

where $s = (m+1)(m+2)(m+3)$.

$$Q(z,t) = \int_0^H u(z,r,t)\ dr. \tag{16}$$

$$\bar{Q} = q + 1. \tag{17}$$

Eq. (15) is integrating in respect of zover onewavelength to obtain the pressure drop

$$\Delta p = \int_0^1 (-\frac{\partial p}{\partial z})\, dz$$
$$= \int_0^1 p \; dz. \tag{18}$$

IV. RESULTS AND DISCUSSIONS

Herschel-Bulkley fluid having non-zero yield stressis examined to know the bloodtransport pattern inanon-uniform tube having elasticity. The yield stress considered, accompanied bythe power law index, helps in exposing the shear thinningnature and thus inferring the blood flow characteristics.

The characteristics of the non uniform tube are observed by the graph of flow rate against pressure drop (Eq.18) using Mathematica software.

Pressure difference as plotted against$\bar{Q}$ for varying values of Darcy number is represented in Fig. (1.2). decrease in the pressure drop$\Delta p$is observedwith increasing Darcy number Da in pumping region and the oppositeoutcome is seen in the region of co-pumping.

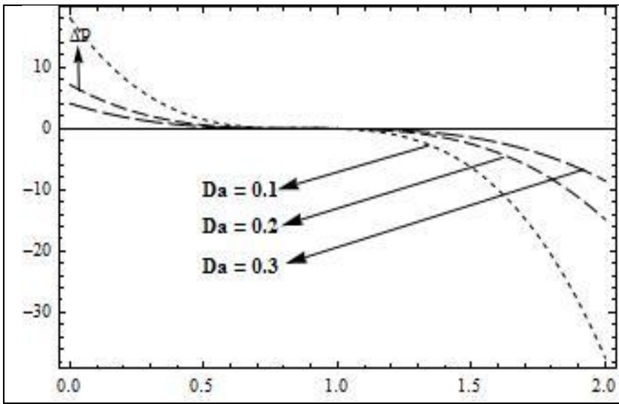

Fig.1.2.Outcomes of Darcy parameter.

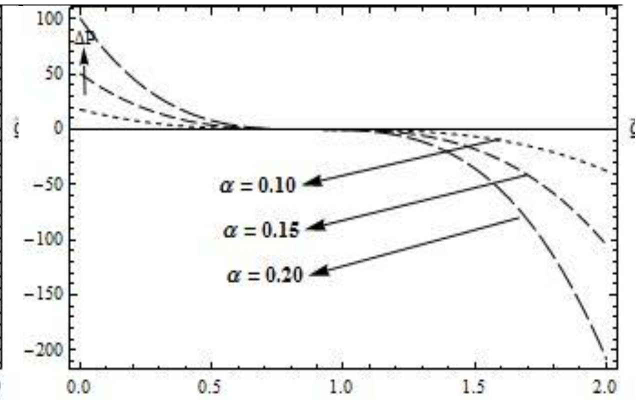

Fig.1.3.Outcomes of slip parameter.

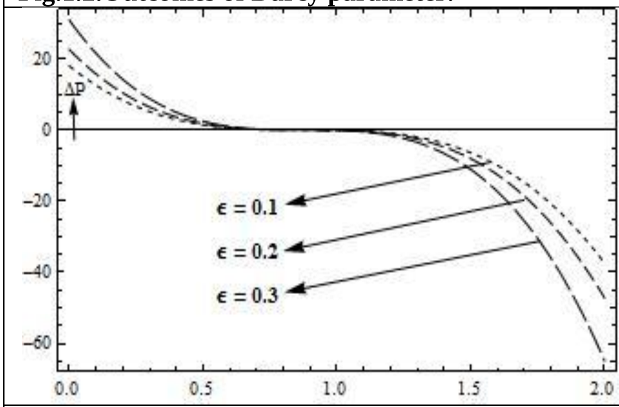

Fig.1.4.Outcomes of porous thickening of the wall.

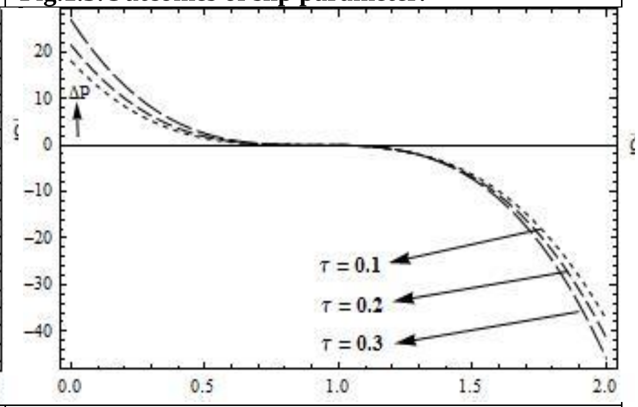

Fig.1.5.Outcomes ofyield stress parameter.

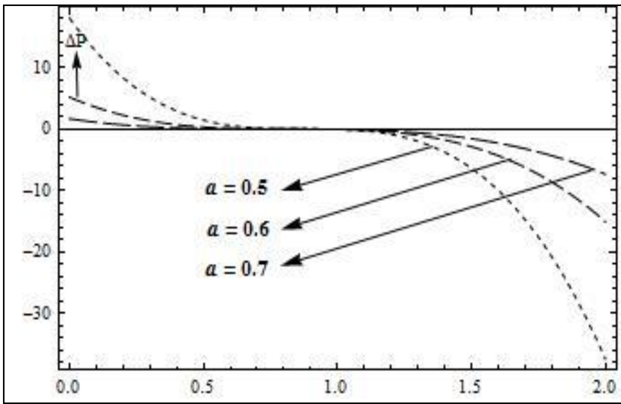

Fig.1.6: Outcomes of radius of tube.

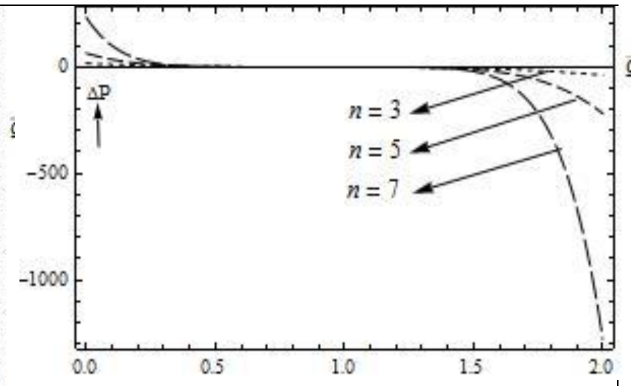

Fig.1.7: Outcomes of power law index.

From Fig. (1.3) it is inferredthat pressure dropΔpincreases with increasing slip parameter α in pumping expanse and the contradictory result is seen in the co-pumping expanse.

It is seen from Fig. (1.4) that as the porous wall thickens the pressure dropΔpalso increases in pumping expanse while effect is reversed in the co-pumping expanse.

The pressure differencevariation with time averaged flow rate is consideredwithvariations inτas in Fig. (1.5). Increase in the pressure drop$\Delta p$is seen with increasing yield stress τ in the region of pumping while effect is contradictory in the co-pumping region.Observations infer that the peristaltic actionon the tube wall, pumps with a larger pressure,than a power law index,because of the existence of plug flow region in Herschel-Bulkley fluid.

Fig. (1.6) depicts decrease in the pressure drop$\Delta p$with increasing radius of the tube in pumping expanse and effect is opposite in the co-pumping expanse.

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Increase in the power law index$n$increases the pressure riseΔp in pumping expanse and the opposite outcome is seen in the co-pumping expanse as depicted in Fig. (1.7).

V. CONCLUSION

The Herschel-Bulkleyfluid model is analyzed for the transport induced by sinusoidal peristaltic waves with small Reynolds number. Pressure drop increases with the power law index, slip parameter, yield stress and porous thickening of the wall but decreases with radius of the tube and Darcy number**.**

The above analysisprovidesacceptable results that signify some of the natural phenomena, mainly the flow of blood in arteries which can be processed and handled in case of dysfunction.The Herschel-Bulkley fluid is more emphasizedas, blood behaves similar to Herschel-Bulkley fluid rather than power law and Bingham fluids, thus making itappropriate in the analysis of blood and other physiological fluid flows stimulated by peristalsis.

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